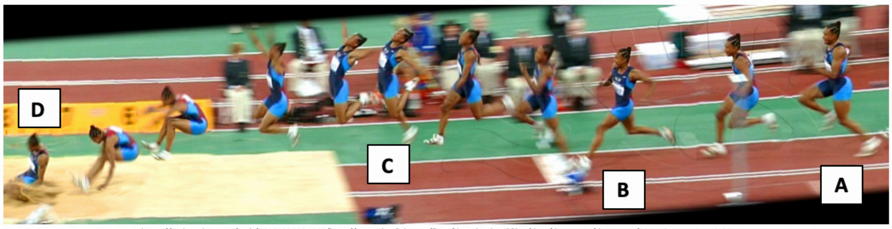
1. (8 points) Below is a multi-flash photo of an athlete who runs up to a line before jumping (long jump). She pushes her foot off the ground running at A, steps on the jump line at B, is mid-air at the highest point of her jump at C, and sits on the sand at D. The time interval between each photo is 0.1 seconds. The distance she travels, from A to D, is 5 meters.



https://web.archive.org/web/20101228234449/http://www.dartfish.com/floor/download.cgi?file=/data/document/document/22.jpg&name=Long%20jump

a. (2 points) Sketch a 2D motion diagram for this multi-flash photo, from A to D.

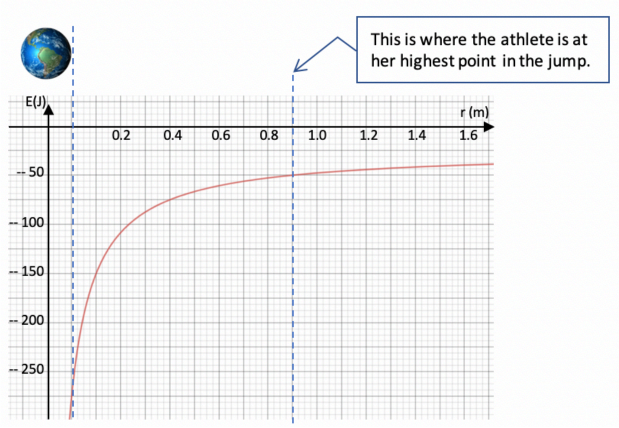
criteria	points
Sketches dots to represent center of mass for each flash photo, one dot for each flash photo. (Reversed motion diagrams accepted.)	0.5
Increasing/constant spaces between dots from A-B, decreasing spaces between dots from B-C, and increasing spaces between dots from C-D.	1
Includes arrow to represent speed for each dot. (Arrows may go from right to left, or left to right.)	0.5

~~b. (0 points) Sketch a velocity vs. time graph for the 2D motion diagram.~~

c. (3 points) Sketch a free body diagram for A, B, and C.

criteria			points
A	Force(s) are sketched with correct orientation and includes:	$\vec{F}_g, \vec{F}_N, \vec{F}_{SF}, \vec{F}_D$	1
B	Force(s) are sketched with correct orientation and includes:	$\vec{F}_g, \vec{F}_N, \vec{F}_{SF}, \vec{F}_D$	1
C	Force(s) are sketched with correct orientation and includes:	$\vec{F}_g, \vec{F}_D$	1

d. (3 points) The potential energy graph for the athlete, starting from C ($h = 0.9$ m), is shown below.



If her mass is 60 kg and the horizontal component of her velocity at D is 8 m/s, what is her speed right before she arrives at D?

criteria		points
Any statement indicating conservation of energy	$\sum E_{\text{initial}} = \sum E_{\text{final}}$ $\quad K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}}$	0.5
Subtracts correct values from graph	$\Delta E = -50\text{ J} - (-275\text{ J}) = 225\text{ J}$	0.5
Correct set-up and calculation for vertical component of final velocity	$K_{\text{final}} = \Delta E$ $\frac{1}{2}(m)\lvert\vec{v}_{\text{final,y}}\rvert^2 = 225\text{ J}$ $v_{\text{final,y}} = 2.74\text{ m/s}$	1
Addresses horizontal component in calculation of final speed	$\lvert\vec{v}_{\text{final}}\rvert = \sqrt{(v_{\text{final,y}})^2 + (8\text{ m/s})^2}$ $\mathbf{\lvert\vec{v}_{final}\rvert = 8.46\text{ m/s}}$	1

2. (12 points) A car of mass 1,000 kg moving at constant speed of 12 m/s is involved in a head-on, completely inelastic, collision with an object of mass M at time t = 0. This object was initially at rest. Shortly after the collision, the speed v (in m/s) of the car-object system can be represented as a function of time t (in seconds) by this expression:

$$v = \frac{8}{1+5t}$$

a. (2 points) Sketch and label the relevant components/kinematics of the system **before** and **after** the collision.

criterion			points
Before collision	Clearly sketched and labeled mass, and velocity, for each object	$m_1 = 1000\ \text{kg} \quad M = ?\ \text{kg}$ $v_{1,\text{initial}} = +12\frac{\text{m}}{\text{s}} \quad v_{M,\text{initial}} = 0\ \text{m/s}$	1
After collision	Clearly sketched and labeled mass, and velocity, for each object	$m_{\text{final}} = m_1 + M \quad v_{\text{final}} = \frac{8}{1+5t}\frac{\text{m}}{\text{s}}$	1

b. (3 points) Calculate the mass M of the object.

criterion		points
Any statement indicating conservation of momentum	$\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}} \quad m_1\vec{v}_{1,\text{initial}} = (M+m_1)\vec{v}_{\text{final}}$	1
Correct substitution of quantities for $\sum \vec{p}_{initial}$ and $\sum \vec{p}_{final}$	$\sum \vec{p}_{\text{initial}} = m_1\vec{v}_{1,\text{initial}} = (1000\ \text{kg})\left(12\frac{\text{m}}{\text{s}}\right)$ $\sum \vec{p}_{\text{final}} = (M+m_1)\vec{v}_{\text{final}} = (M + 1000\ \text{kg})\left(\frac{8}{1+5t}\frac{\text{m}}{\text{s}}\right)$	0.5
Correct set-up for calculation	$(1000\ \text{kg})\left(12\frac{\text{m}}{\text{s}}\right) = (M+1000\ \text{kg})\left(\frac{8}{\mathbf{1+5t}}\frac{\text{m}}{\text{s}}\right)$ Since collision occurs right after t = 0, then $(1000\ \text{kg})\left(12\frac{\text{m}}{\text{s}}\right) = (M+1000\ \text{kg})\left(8\frac{\text{m}}{\text{s}}\right)$	1
Correct answer	$M = 500\ \text{kg}$	0.5

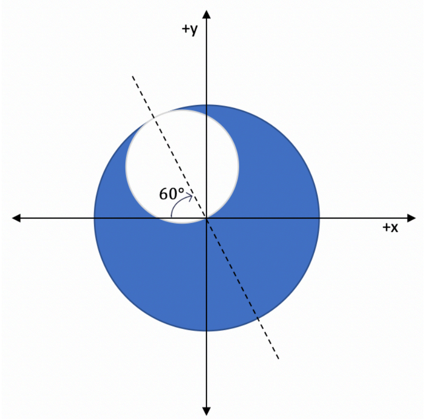
c. (4 points) Determine an expression for the resisting force ($\vec{F}_r$) on the car-object system after the collision as a function of time t.

Solution 1	Solution 2	points
States Newton's Second Law $\vec{F} = m_{\text{final}}\vec{a}$	Indicates force is time derivative of momentum $\vec{F} = \frac{d\vec{p}}{dt}$	1
Any indication that acceleration is time derivative of velocity $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{8}{1+5t}\right)$	Any expression of force with respect to derivative of velocity $\vec{F} = m_{\text{final}}\frac{d\vec{v}}{dt} = m_{\text{final}}\frac{d}{dt}\left(\frac{8}{1+5t}\right)$	1
Correct set-up for derivation $\vec{a} = -\frac{40}{(1+5t)^2}$ $\vec{F} = (1500)\left(-\frac{40}{(1+5t)^2}\right)$	Correct set-up for derivation $\frac{d}{dt}\left(\frac{8}{1+5t}\right) = -\frac{40}{(1+5t)^2}$ $\vec{F} = (1500)\left(-\frac{40}{(1+5t)^2}\right)$	1
Correct answer (any form of magnitude and direction accepted in final answer) $\vec{F} = -\frac{60{,}000}{(1+5t)^2}\hat{x}$		1

d. (3 points) Assuming an initial position of x = 0, determine an expression for the position of the car-object system after the collision as a function of time t.

criterion		points
Any indication that velocity is time derivative of position	$\vec{v} = \frac{dx}{dt} = \frac{8}{1+5t}$	0.5
Correctly expresses equation as integral, with/without limits	$x = \int \frac{8}{1+5t} dt$	1
Correct set-up for derivation	Substitutes $u = 1 + 5t$ and $du = 5dt$. $x = \frac{8}{5}\int \frac{du}{u} = \frac{8}{5}\ln(1+5t) + C$ Since x = 0 when t = 0, then C = 0.	1
Correct answer	$x = \frac{8}{5}\ln(1+5t)$	0.5

3. (6 points) A circular disc (uniform mass, radius of 2R) has a circular hole (radius R) cut out of it; the hole is tangent to the edge of the circular disc, as shown below.



Using the displayed coordinate system, what is the position vector for the object's center of mass?

criteria		points
Indicates center of mass lies along dotted line.		0.5
Treats hole as unknown variable to be solved in center of mass equation	$z_{\text{CM,object+hole}} = \frac{z_{\text{CM,object}} m_{\text{object}} + z_{\text{CM,hole}} m_{\text{hole}}}{m_{\text{object}} + m_{\text{hole}}} = 0$	1
Correct set-up for calculation of center of mass	$z_{\text{CM,object}} = -\frac{z_{\text{CM,hole}} m_{\text{hole}}}{m_{\text{object}}} = \frac{R\, m_{\text{hole}}}{m_{\text{object}}}$	1
Correct calculation of masses	$m_{\text{hole}} = \pi R^2 \rho$ $m_{\text{object}} = \pi(2R)^2\rho - m_{\text{hole}} = \pi(2R)^2\rho - \pi R^2 \rho = 3\pi R^2 \rho$	1
Correct solution for the center of mass for the object	$z_{\text{CM,object}} = \frac{R\, m_{\text{hole}}}{m_{\text{object}}} = \frac{R(\pi R^2 \rho)}{3\pi R^2 \rho} = \frac{R}{3}$	1
Trigonometric functions used to determine the position vector	$x = \frac{R}{3}\cos 60 = \frac{R}{6}$ $\quad y = \frac{R}{3}\sin 60 = \frac{R\sqrt{3}}{6}$	1
Correct answer, which includes signs for each vector component	$r = \begin{bmatrix} \frac{R}{6} \\ -\frac{R\sqrt{3}}{6} \end{bmatrix}$	0.5
Alternate responses, based on symmetry, may be used to determine the center of mass.		